

ATTACHING THEORIES OF CONSCIOUSNESS TO BOHMIAN QUANTUM MECHANICS *

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Abstract

The de Broglie-Bohm theory of quantum mechanics (here simply called Bohmian Mechanics or BM) [1-10] is an augmentation of “bare” quantum mechanics (the bare theory being given by an algebra of operators and a quantum state that sets the expectation values of these operators) that includes a definite history or Bohmian trajectory. This definite trajectory gives BM a somewhat more classical flavor than most other forms of quantum mechanics (QM) (though the trajectory certainly has highly nonlocal and other nonclassical aspects in its evolution), but to see whether or not this makes a difference for observations by conscious beings, one needs to attach theories of conscious perceptions to BM and other forms of QM. Here I shall propose various forms of theories of consciousness for BM, which I shall call *Sensible Bohmian Mechanics* (SBM), and compare them with a proposal I have made for a theory of consciousness attached to bare QM, which I call *Sensible Quantum Mechanics* (SQM) [11-15]. I find that only certain special forms of SBM would give essentially similar predictions as SQM, though a wider class might be in practice indistinguishable to any single observer. I also remain sceptical that a viable complete form of SBM will turn out to be as simple a description of the universe as a viable complete form of SQM, but of course it is too early to know yet what the form of the simplest complete theory of our universe is.

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I should explain at the outset that my attempt to incorporate consciousness within physics is nonstandard, since most physicists would probably consider this attempt premature, or simply not part of physics. I agree that it *is* probably premature to try to give a complete detailed theory of consciousness, but since we have developed mathematical frameworks like mechanics for describing other aspects of the universe, even though we do not yet know the correct complete detailed form of the mechanics describing our universe, it surely would be helpful to try to develop a mathematical framework for describing consciousness, even though we are a very long way from a correct complete detailed theory for it. The objection that consciousness is not part of physics may be historically valid as a sociological analysis of what most physicists actually study (for the reason, I would guess, that physics concentrates on the simplest fundamental descriptions of aspects of our universe, and so far consciousness does not seem to be simple enough to be included in physics). However, physics has historically continued to be extended to describe a broader and broader range of aspects of our universe, so that in some branches of physics there is even talk today of ‘theories of everything.’ Thus it seems plausible to me that physics should eventually attempt to describe consciousness itself. Certainly talk of ‘theories of everything’ in physics seems rather hollow if physics is required to refuse to consider consciousness.

Furthermore, physics, unlike mathematics, is generally seen to be rooted in experiment and observations, and these fundamentally come down to conscious experience. This is particularly true in quantum mechanics, where in many formulations observations are crucial. If one refuses to consider conscious observations, then there seems to be nothing wrong or lacking in bare QM, though then it is just a beautiful mathematical theory for an unconscious world that is totally divorced from observations. However, precisely at the stage at which one wants to explain what is consciously observed, one needs at least some glimpse of a framework for connecting consciousness to observations. Under the assumption that classical mechanics (CM) correctly described our universe, many physicists usually imagined, I suspect, some simple idealized form of psycho-physical parallelism in which the content of one’s conscious perception is very similar to certain aspects of the configuration of the classical universe. Then, say, if one looked at a clock whose hands were at the 12 o’clock position, one would have a conscious perception that the clock read 12 o’clock.

One of the problems of ordinary QM is that this simple form of psycho-physical parallelism that seemed adequate in CM does not work in such a naïvely straightforward way. Even though many physicists do not explicitly wish to consider any theories of consciousness, I suspect that much of the trouble they have with QM arises from the fact that they actually implicitly have something like this simple form of psycho-physical parallelism sketched above, and then they find difficulty

fitting it with QM. This is indeed one of the main motivations for BM, since it has a definite trajectory, with a definite position in configuration space (e.g., definite particle positions) at each time, to which the simple form of psycho-physical parallelism can readily be attached. Since the sketch of a theory of consciousness is probably implicit in many physicists' assumptions, one does not need to refer to it explicitly when one extols the merits of a definite trajectory in BM, which one can thus do without making other physicists uncomfortable by mentioning the subject of consciousness that physicists do not really understand very well yet.

However, since Sensible Quantum Mechanics [11-15] gives a glimpse of a framework for a psycho-physical parallelism to the quantum world that appears to be just as adequate (though of course just as sketchy in detail) as the simple form implicitly assumed for a classical world, one can see at least this one general possibility for overcoming the problem that many physicists have in relating QM with conscious observations (a problem made worse by a reluctance to consider consciousness explicitly and to acknowledge one's naïve preconceptions about it). SQM seemed extremely obvious when I hit upon it, so initially it was surprising to me that it had not been developed many years ago (though I did later find that Lockwood [16] had a few years ago expressed highly concordant ideas in less mathematical form). It occurred to me that the novelty of SQM was perhaps simply because of physicists' reluctance to consider consciousness explicitly and because of the strong psychological hold on their assumptions of the simple form of psycho-physical parallelism possible for the classical world.

In view of the apparent success of SQM in sketching how QM may be combined with consciousness with virtually none of the traditional interpretive problems (at least in my own eyes; most physicists I have talked to still seem to think that any theory of consciousness is either unnecessary or premature), it may be of interest to develop also an SBM theory to combine BM in a similar way with consciousness. Then one can have two similar frameworks, containing consciousness explicitly rather than merely implicitly as has unfortunately too often been done, for comparing ordinary QM with BM. Since in my nonstandard view I consider SQM (which is the bare QM of an algebra of operators and a quantum state, with no measurement hypothesis or collapse of the wavefunction, augmented by a theory of consciousness connected to the bare QM by a nonclassical form of psycho-physical parallelism) to be superior to all other forms of ordinary QM, I shall not bother comparing my SBM extension of BM to other forms of QM, but only to what in my biased opinion I consider to be the best form of ordinary QM with consciousness, SQM.

Since BM is even at the unconscious level an augmentation of bare QM, it would be simplest to describe first SQM and then show how it can be augmented to SBM. SQM is given by the following three basic postulates or axioms [13]:

Quantum World Axiom: The unconscious "quantum world" Q is completely described by an appropriate algebra of operators and by a suitable state σ (a positive linear functional of the operators) giving the expectation value $\langle O \rangle \equiv \sigma[O]$ of each

operator O .

Conscious World Axiom: The “conscious world” M , the set of all perceptions p , has a fundamental measure $\mu(S)$ for each subset S of M .

Quantum-Consciousness Connection: The measure $\mu(S)$ for each set S of conscious perceptions is given by the expectation value of a corresponding “awareness operator” $A(S)$, a positive-operator-valued (POV) measure [17], in the state σ of the quantum world:

$$\mu(S) = \langle A(S) \rangle \equiv \sigma[A(S)]. \quad (1)$$

Here the Quantum World Axiom is the basic axiom of the mechanics of bare QM, and the Conscious World Axiom is the basic postulate I shall make about the conscious world, even when I go from SQM to SBM. Perhaps I should remind the reader [13] that a perception p is in this context taken to mean the entirety of a single conscious experience, all that one is consciously aware of or consciously experiencing at one moment, the total “raw feel” that one has at one time, or [16] a “phenomenal perspective” or “maximal experience.”

Besides a modification of the Quantum World Axiom in BM, in SBM I shall also modify the Quantum-Consciousness Connection, though in both SQM and SBM I shall assume that the measure for each set of conscious perceptions is given by some functional of the corresponding quantum or mechanical world. In SQM the measure is a functional of the operators and the quantum state, and I made the simplest assumption that this functional is linear in the quantum state and is the expectation value of a particular operator (an “awareness operator” $A(S)$) for each set S of perceptions. Of course, one could readily contemplate generalizations in which the measure is a *nonlinear* functional of expectation values [13], but I shall not do that here.

Bohmian mechanics is most simply given in the case in which a quantum state is given by a time-dependent wavefunction over some configuration space (say, for simplicity, of particle positions). One augments this quantum state by a history or time-parametrized trajectory in this configuration space with the velocity vector at each point being given by a suitable functional of the wavefunction (a normalized gradient of its phase for spinless particles) [1-10]. This gives a first-order differential equation for the trajectory, so it is uniquely determined by this equation (which is itself uniquely determined by the wavefunction except at its zeros, but those isolated points do not cause any trouble) and by the position of the trajectory at any one (e.g., initial) time.

Because BM thus has a trajectory as well as a quantum state, there are in principle more things on which the measure for conscious perceptions can depend. Again assuming a linear (if nontrivial) dependence on the quantum state, I shall propose that Sensible Bohmian Mechanics (SBM) is given by the following three axioms:

Bohmian World Axiom: The unconscious “Bohmian world” B is described by a normalized time-dependent Hilbert-space wavefunction $\psi(x, t)$ (obeying an ap-

appropriate Schrödinger equation that determines the wavefunction at all times t once it is given at one time) on a configuration space with coordinates x (indices suppressed) and by a Bohmian trajectory T in this configuration space whose velocity is given by a suitable functional of the wavefunction.

Conscious World Axiom: The “conscious world” M , the set of all perceptions p , has a fundamental measure $\mu(S)$ for each subset S of M .

Bohmian-Consciousness Connection: The measure $\mu(S)$ for each set S of conscious perceptions is given by the expectation value of a corresponding “awareness operator” $A(S, T)$, a positive-operator-valued (POV) measure [17] that depends on the Bohmian trajectory T , in the wavefunction of the Bohmian world:

$$\mu(S) = \int dx \psi^*(x, t_0) A(S, T) \psi(x, t_0). \quad (2)$$

(Here the integral can be taken at any time t_0 , since the Schrödinger equation determines the wavefunction at all times from its form at time t_0 , though of course the form of $A(S, T)$ would depend on the particular t_0 chosen. Also, for simplicity, I am suppressing possible spin indices on the wavefunction as well as the coordinate indices on the x that denotes a point in the multi-dimensional configuration space.)

As in SQM [13], so in SBM it is convenient to hypothesize that the set M of all possible conscious perceptions p is a suitable topological space with a prior measure

$$\mu_0(S) = \int_S d\mu_0(p). \quad (3)$$

Then, just as for SQM, the linearity of positive-valued-operator measures over sets allows one to write the awareness operators for SBM as

$$A(S, T) = \int_S E(p, T) d\mu_0(p), \quad (4)$$

a generalized sum or integral of SBM “experience operators” or “perception operators” $E(p, T)$ for the individual perceptions p . Similarly, one can write the measure on a set of perceptions S as

$$\mu(S) = \int_S d\mu(p) = \int_S m(p) d\mu_0(p), \quad (5)$$

in terms of a measure density $m(p)$ that is the quantum expectation value of the experience operator $E(p, T)$ for the same perception p :

$$m(p) = \int dx \psi^*(x, t_0) E(p, T) \psi(x, t_0). \quad (6)$$

It is simplest to consider the two extreme cases in which, for each S or p , $\mu(S)$ or $m(p)$ depends only on the wavefunction $\psi(x)$ or on T . In the first of these cases, one essentially has a form of SQM with the quantum world consisting of a state that has the particular form of a time-dependent Hilbert-space wavefunction on some

configuration and of operators on such wavefunctions, and with the Bohmian world consisting of this quantum world augmented by the Bohmian trajectory T . However, in this case the Bohmian trajectory has absolutely no effect on the conscious world with its perceptions and measures. Since our perceptions are the only direct contact we have with the world, if they are completely unaffected by the Bohmian trajectory, there would then seem to be no motivation to add them to one's theory of the quantum world. Although it indeed fits my own prejudice not to augment bare QM for the unconscious aspects of the world and thus not to bother with any Bohmian trajectory in theories of the quantum world that I prefer, for the purpose of the present discussion on BM and its extension to SBM it would seem better to consider a Bohmian-Consciousness Connection in which the Bohmian trajectory really does have an effect on the measure for sets of perceptions. As discussed above, a motivation for the trajectory in BM is that its point in configuration space at each time seems closer to the content of conscious perceptions than does the quantum state or wavefunction itself.

The second extreme possibility is that for each S or p , $\mu(S)$ or $m(p)$ depends only on the trajectory T . This can be accomplished in Eqs. (2) or (6) by having $A(S, T)$ or $E(p, T)$ not have any nontrivial dependence on x but simply be an S - or p -dependent numerical function purely of T , say $A(S, T) = a(S, T)$ (i.e., a function purely of S and of T times the identity operator in the wavefunction Hilbert space, so for a normalized wavefunction, one gets $\mu(S) = a(S, T)$), or, say, $E(p, T) = e(p, T)$ (so then $m(p) = e(p, T)$).

In this case one might say that he could simply dispense with the wavefunction $\psi(x, t)$, since the measure for one's perceptions would then depend only on the Bohmian trajectory T . However, the time dependence of this trajectory depends in a simple way upon the wavefunction, so the latter could still be a useful element in giving the simplest description of the trajectory upon which the measure for perceptions depends. (Similarly, one could in principle dispense with the quantum or Bohmian world altogether and consider merely the existence of the conscious world, or actually even only one's own present conscious perception within it, as that is all he has direct experience of, but the description of even just one's own present conscious perception by itself may be simpler if one postulates a whole conscious world with many measured sets of perceptions, and a description of this conscious world by itself may also be simpler if one postulates the existence of a quantum or Bohmian world on which it depends. I personally also believe that a description of the full SQM or SBM world would be simpler if one postulates the existence of an omniscient, omnipotent God as the Creator of this world, but of course this further extrapolation from direct conscious experience takes one beyond what is traditionally called physics to metaphysics.)

If for each S and p , $A(S, T)$ and $E(p, T)$ are the purely numerical functions (or functionals) $a(S, T)$ and $e(p, T)$ of the trajectory T , with no dependence on the wavefunction, there is still the question of what kind of functions of T they

are. In principle they could be nonlocal functionals of the entire trajectory, so that one's perception depended on the entire Bohmian history. The fact that we do not normally perceive much detail about the future might be taken to suggest that there is a time $t(p)$ associated with each perception, and that $e(p, T)$ depends only on the part of the trajectory at times at and earlier than $t(p)$. The dependence on the part of T at times earlier than $t(p)$ might then be postulated to give the memory components of perceptions.

However, the fact that drugs and physical conditions such as Alzheimer's disease seem to have a strong effect on memory suggests to me that it is more plausible to assume that the memory components of present perceptions are directly caused by present physical conditions (perhaps having something to do with neural connections in the brain) rather than being caused directly by past conditions such as the past part of the Bohmian trajectory. Thus even if the measure density $m(p)$ for a perception p were determined entirely by the Bohmian trajectory T rather than by it and by the quantum state (or entirely by the quantum state, as in SQM, which I personally prefer), I would think it to be the simplest plausible assumption that $m(p)$ is determined by the trajectory at the single time $t(p)$, which can be considered to be the time of the perception (something not well defined in SQM except in rather *ad hoc* ways [13]). In other words, I am claiming that it seems simplest to suppose that each conscious perception is directly only of the present, though that present can include the records that produce the perception components of memory.

Of course, it may be an extreme idealization to suppose that the dependence is on precisely that single time rather than being spread out over, say, one Planck time. But I see no evidence that each of our perceptions need be directly affected by anything in the unconscious quantum or Bohmian world spread out over a time long in, say, seconds. Although a second is admittedly very long compared with the Planck time, it is certainly much shorter than cosmological times, and there is no evidence that I see that puts a lower limit on the time over which the unconscious quantum or Bohmian world affects a single conscious perception (unless one adopts a theory in which instants and time periods less than some lower limit simply do not exist).

Even if the measure density $m(p)$ for a perception p is determined entirely by the Bohmian trajectory at the instant $t(p)$, there is still the question of what aspect of the trajectory at that instant determines $m(p)$. In principle, it could be determined by the position, velocity, acceleration, and/or higher time derivatives of the position in the configuration space x as a function of time t , evaluated at the time $t(p)$.

If one wants to get the closest agreement between an SQM theory and a version of SBM with $m(p)$ depending purely on the Bohmian trajectory, it seems that one should select the following features in one's choice of the SQM and the SBM:

- (1) The SQM and SBM should have the same bare quantum world, a normalized time-dependent Hilbert-space wavefunction on some configuration space.
- (2) The position x_0 of the Bohmian trajectory T at time t_0 should be chosen ran-

domly with the probability distribution given by the quantum “probability” density $\psi^*(x, t_0)\psi(x, t_0)$. (The Bohmian equation for the velocity of the trajectory as the gradient of the phase of the wavefunction ensures that if one selects a continuum ensemble of trajectories such that at time t_0 their measure density in the configuration space is the quantum “probability” density $\psi^*(x, t_0)\psi(x, t_0)$ at that time, then the measure density of this ensemble of trajectories is carried forward in time such that at time t it is precisely the quantum “probability” density $\psi^*(x, t)\psi(x, t)$ at that new time t . In this sense the “random” probability distribution $\psi^*(x, t)\psi(x, t)$ preserves its functional dependence on the wavefunction $\psi(x, t)$ at all times.)

(3) The SQM should have each of its experience operators $E(p)$ being a projection operator $P(s(p), t(p))$ onto a subset $s(p)$ of the configuration space and at a time $t(p)$ that both depend on the perception p .

(4) The SBM should have each of its experience operators being of the form $E(p, T) = e(p, T)$ and having a nonzero constant value (say 1) if the trajectory T at the time $t(p)$ (the same function of the perception p as in the corresponding SQM theory) is within the subset $s(p)$ of the configuration space (the same subset for each p as in the corresponding SQM theory), and having the value zero if the trajectory at time $t(p)$ is not within $s(p)$.

Even in this idealized correspondence between SQM and SBM theories, one will not get for each the same measure density $m(p)$ for individual perceptions or the same measure $\mu(S)$ for sets of perceptions. However, if one enlarged the BM and SBM theories to *Continuum Bohmian Mechanics* (CBM) and *Sensible Continuum Bohmian Mechanics* (SCBM) theories respectively that are hereby defined to include a whole continuum ensemble of trajectories, with their measure density given by $\psi^*(x, t)\psi(x, t)$ at any time t (whose preservation at all times is guaranteed by the Bohmian equation of evolution for each trajectory in the ensemble), then one would get agreement if one integrated, over the measured continuum ensemble of all trajectories, the $m(p)$, and hence also the $\mu(S)$, that one gets from each trajectory in the particular SBM theory with its own individual trajectory. In contrast, for a particular SBM theory with its own particular trajectory T , one would get a zero measure for any set S of perceptions p which all give configuration-space subsets $s(p)$ that the particular trajectory T is not in at the corresponding times $t(p)$. (On the other hand, any set of perceptions with nonzero measure in any particular SBM theory would necessarily have a nonzero measure in the corresponding SQM theory when features (1)-(4) hold for the pair of corresponding theories.)

Nevertheless, the fact that each perception p that actually occurs (i.e., that has a nonzero measure density) has no direct awareness of any other perception means that one cannot absolutely test whether or not one is in an SBM universe in which another perception p' , with the particular trajectory T not in the configuration-space subspace $s(p')$ at the time $t(p')$ and hence with $m(p') = 0$, does not actually occur, or whether one is in the corresponding SCBM or SQM universe in which $m(p') > 0$ so that this other perception p' does actually occur. It seems that perhaps the best one

can do is to compare what the corresponding SBM and SQM theories give for the *typicality* of the perception p (the fraction of the measure of other perceptions p' that have a measure density not greater than that of p , or some modification of this such as the *dual typicality* [13]). Although if the features (1)-(4) proposed above hold, the corresponding SCBM and SQM theories would give the same typicality for each perception, even the typicality for an existing perception in the SBM theory and that for the same perception in the corresponding SQM theory would differ. However, one might hope that for a reasonably large fraction of individual Bohmian trajectories, chosen randomly out of the continuum ensemble with the $\psi^*(x,t)\psi(x,t)$ measure density, most of the measure of the actually existing perceptions in the resulting particular SBM theory would occur for a set of actually existing perceptions whose typicalities would not be too low in either that particular SBM theory or in the corresponding SQM theory. If this were true (and its truth might well depend on the form of the underlying bare quantum theory, i.e., on the wavefunction, so that this could well be a question worth further investigation), then the typicality of most existing perceptions would not give a strong test between a particular SBM theory that predicted its existence and the corresponding SQM theory.

The fact that one's perception p cannot absolutely rule out any SBM or SQM theory that gives it a nonzero measure density $m(p)$, and the fact that one can apparently only use something like the typicality of the perception (interpreted as the likelihood or conditional probability for the perception given the theory) to weight the prior probability assigned to the theory in a Bayesian analysis to get the posterior probability for the theory [13], means that even if one has a pair of SQM and SBM theories that do not share the features (1)-(4) above, one's actual perception will not necessarily rule out either of these theories. But, crudely speaking, the greater the degree to which the features (1)-(4) are not held by the pair of theories, the greater the possibility apparently is that the typicalities assigned by the two theories to one's perception will differ sufficiently greatly that one can use them to deduce a very low posterior probability to one or the other of the pair of theories.

Of course, I should emphasize that the "one" who is postulated to be doing the comparison of typicalities must be one who can indeed calculate them from the SQM and SBM theories in question. In practice this might be possible only for a being of such intelligence that he, she, or it (or whatever is the correct pronoun for such a being that may not have any sex and yet is more intelligent than we usually ascribe to "things") can exist only outside our universe. Those of us within the universe might be expected to be able to get only a very crude estimate of such typicalities, but if one can even see roughly that they differ by many orders of magnitude between two theories, that would be sufficient to get a reasonably good idea of which theory to reject (except in the case in which the prior probabilities differed by roughly the same number of orders of magnitude in the opposite direction).

There is also the apparently completely subjective question of what prior probabilities to assign different SQM and SBM theories before weighting them with the

typicalities of one's perception in a Bayesian analysis to get the posterior probabilities for the theories. (I say "apparently," since conceivably there is an actual existing measured set of different universes, each described by a different SQM or SBM theory, so that the actual measure of these universes gives an ontologically objective prior frequency-type probability to the different theories describing the different universes, but since we certainly do not have access to all of these conceivable universes or know their measure even if it does exist objectively, for us epistemologically the prior measure must surely be subjective.) Here I should lay my cards on the table and explain the prejudices that I have against assigning BM and SBM theories high prior probabilities.

First, I should say that I would prefer to assign higher prior probabilities to simpler theories. Perhaps most physicists would agree, but they might differ on how to weight or even how to rank the simplicity of different theories. (I suspect that this may lie at the core of more metaphysical disagreements as well, such as whether theism or atheism is true.)

I myself think that adding the Bohmian trajectory to bare quantum mechanics reduces the simplicity and thus the prior probability I would assign to the theory. Perhaps for nonrelativistic QM, the extension to CBM with its continuum of trajectories that have the $\psi^*(x, t)\psi(x, t)$ measure density is not too great a loss of simplicity, since this ensemble of trajectories does not take too much additional information to specify and has certain nice properties (such as the measure density's remaining $\psi^*(x, t)\psi(x, t)$ for all time). However, to pick out a single trajectory from this ensemble for a particular BM (as opposed to CBM) theory would seem to require much more *ad hoc* information. I can think of certain choices that do not require too much additional information, such as choosing the trajectory whose time integral of $\psi^*(x, t)\psi(x, t)$ evaluated along the trajectory itself is the maximum out of all possible Bohmian trajectories, but even these relatively simple choices seem rather *ad hoc* and ugly.

Incidentally, I might point out that one could have theories of *Generalized Continuum Bohmian Mechanics* (GCBM), and its augmentation of *Sensible Generalized Continuum Bohmian Mechanics* (SGCBM) to include the Conscious World Axiom and the (suitably generalized, as discussed above for SCBM) Bohmian-Consciousness Connection, in which there is a continuum of Bohmian trajectories with a measure density that is a general normalized nonnegative function over the configuration space at any one time and which is transported to other times by the Bohmian equation for the velocity of the trajectories. One could then say that CBM is the special case (probably the simplest) in which this measure density is $\psi^*(x, t)\psi(x, t)$, and ordinary BM with its single trajectory is the special case in which the measure density at any one time is a delta-function distribution (not so simple, as discussed above, since one must specify the location of this delta-function at one time.) However, even within the wide set of possibilities of GCBM or SGCBM theories, I do not see any so simple as simply leaving out the trajectories altogether, as in bare

QM and SQM theories.

Now a supporter of BM might object that although BM is almost certainly more complicated than bare QM in requiring the extra element of the Bohmian trajectory, in another way it is simpler in not requiring any operators on the Hilbert space of wavefunctions (except presumably for the Hamiltonian that generates the evolution by the Schrödinger equation). However, once the Hilbert space of wavefunctions is defined, it takes very little additional information to define the set of operators on this space that maps it into itself. Perhaps this additional information is comparable to that for the continuum ensemble of trajectories in CBM, so I would admit that CBM is of comparable simplicity to bare QM for the same Hilbert space (though specifying a precise single trajectory in ordinary BM seems to me in general more complicated). But even in a comparison with CBM, the framework of bare QM allows a much greater range of possibilities for the quantum state than the restriction to a wavefunction over configuration space (or some slight generalizations to include spin, etc.) that seems to be necessary for present formulations of BM and CBM, and it might turn out that a quantum state that cannot be written as a wavefunction will be at the heart of the simplest complete description of the universe.

A supporter of BM might also object that even if BM is indeed more complicated than bare QM in having its Bohmian trajectory, when one attaches a theory of consciousness, the result is simpler if one can attach it to a theory with a trajectory. For example, in the Sensible theories outlined above, a supporter of SBM over SQM might say that BM allows simpler awareness operators $A(S, T)$ (e.g., of the form $a(S, T)$, with no nontrivial dependence on the wavefunction) in the Bohmian-Consciousness Connection of SBM than the awareness operators $A(S)$ allowed in the Quantum-Consciousness Connection of SQM, and that the increased simplicity of $A(S, T)$ over $A(S)$ overbalances the extra complication of the trajectory in SBM. I will admit that it is hard to answer this objection when we are as ignorant as we are about the connection of consciousness to the rest of physics (e.g., about what the awareness operators are in Sensible theories), but I should say that at present I for one am rather sceptical that a viable SBM theory will turn out to be as simple as a viable SQM theory.

Another problem I have with BM theories is that when I imagine applying them to relativistic fields instead of to nonrelativistic particles, I find that even if the quantum state is Lorentz invariant (e.g., the vacuum state), almost all the trajectories are not, and neither is the continuum ensemble of CBM. For example, consider for simplicity a free scalar field. Each plane-wave mode of the field can be considered to be an harmonic oscillator, and the vacuum state of the field can be considered to be a product of the ground states of all of these modes. For such a static state of zero energy, the wavefunction has a constant phase with zero gradient in the configuration space (the space of amplitudes of the modes), so the Bohmian trajectory is static. When one superposes the static amplitudes for the modes, one gets an arbitrary space-dependent static configuration of the scalar field as its Bohmian

trajectory. Except for the special homogeneous cases in which this static configuration is precisely the configuration of constant field everywhere (which in the case of zero field is the one proposed above whose time integral of $\psi^*\psi$ evaluated along the trajectory itself is the maximum out of all possible Bohmian trajectories), these Bohmian trajectories are not Lorentz invariant, for the spatial dependence in the frame in which their evolution was calculated by the Bohmian equation of motion becomes a temporal dependence in any other frame.

Furthermore, in any other frame the trajectory which does not even satisfy the Bohmian equation of motion, so the situation is worse than the version of spontaneous symmetry breaking in which the quantum state is not invariant under the symmetry group of the equations which it solves. In that case the action of an element of the symmetry group on the nonsymmetric solution yields another nonsymmetric solution, but for the inhomogeneous Bohmian trajectories, the action of a Lorentz transformation on them yields time-dependent trajectories that do not even satisfy the Bohmian equations for trajectories. (I am grateful to Shelly Goldstein for pointing out the importance of this distinction.)

Of course, one could say that perhaps Lorentz invariance is just a useful approximation to certain aspects of the world and need not apply to the Bohmian trajectories. After all, in our perceptions there are objects that seem to have fairly definite velocities (with the bulk of nearby ones, such as the earth, being fairly near zero in our local frame) that appear to break Lorentz invariance, so presumably there is nothing blatantly inconsistent with observations to have Bohmian trajectories breaking Lorentz invariance even when the quantum state is Lorentz invariant. However, this lack of Lorentz invariance for all but the homogeneous Bohmian trajectories for a quantum field in the Lorentz-invariant vacuum state, and the need to define a preferred velocity in order to define the Bohmian equations for the trajectories, seems at least aesthetically rather ugly.

I suspect that there would be an even greater degree of ugliness if one attempted to devise a Bohmian version of quantum gravity, say for the quantum cosmology of closed universes. If one has a solution of the Wheeler-DeWitt equation for canonically-quantized general relativity, this gives a wavefunctional of three-geometries that is invariant not only under coordinate transformations of the spatial hypersurface, but also under the equivalent of time translations that are arbitrary at each point of space. Unlike the case of quantum field theory in Minkowski spacetime, in which only one quantum state (the vacuum) has the full Lorentz invariance of the Minkowski spacetime itself, in canonical quantum gravity *every* physical state has the diffeomorphism invariance (including local Lorentz invariance) of general coordinate transformations. It is not clear to me how to construct a Bohmian trajectory for such a theory of quantum cosmology without breaking this diffeomorphism invariance in a complicated and ugly *ad hoc* way.

I would also suspect that if one tries to overcome the severe technical problems of constructing a finite theory of gravity by going from a field theory of general

relativity to a superstring theory, it would be even more cumbersome and ugly to try to construct a Bohmian trajectory. Of course, the bare QM of superstrings at the nonperturbative level is hardly understood at the moment, and virtually nothing is known of how to produce consciousness out of it (say by some form of awareness operators), so one cannot pretend to be certain that this task will be easier without constructing a BM version of superstrings, but I personally suspect that the latter would be simply an onerous burden that is unnecessary. It would seem much more plausible that one should go directly from a bare quantum theory of superstrings (or whatever the ultimate quantum theory is, assuming that it *is* a quantum theory) to a theory of consciousness (say given in terms of the unconscious quantum world by expectation values of awareness operators in SQM), without bothering with trying to find a BM theory with a definite trajectory.

In conclusion, I have outlined how a theory of consciousness may be attached to the Bohmian version of quantum mechanics (BM) to give what I call Sensible Bohmian Mechanics (SBM), in a way highly analogous to the way in which I have proposed attached a theory of consciousness to ordinary quantum mechanics (QM) to give Sensible Quantum Mechanics (SQM) [11-15]. Because BM has a Bohmian trajectory that bare QM does not have, SBM for a fixed wavefunction has more possibilities for awareness operators for giving the measure of sets of conscious perceptions. (Conversely, BM seems to require a wavefunction on a configuration space, or something like it, which QM does not, so in the latter way QM is more flexible.) However, if a SBM is to give mostly typical perceptions that would also be typical in a corresponding SQM theory, it seems that there should be at least four features of the correspondence (listed above), though it is not completely clear that these are entirely sufficient. Although SBM might conceivably allow a simpler set of awareness operators than SQM, this is by no means obvious, and in other respects SBM seems to me to be more complicated and ugly than SQM.

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